JOURNAL OF APPROXIMATION THEORY 26, 192–193 (1979)

Correction to "Metric Entropy of Some Classes of Sets with Differentiable Boundaries"

R. M. DUDLEY

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Communicated by G. G. Lorentz

Received July 21, 1977

In [1], Eq. (3.2) in Theorem 3.1 is incorrect. This equation was not used in the application in [2, Theorem 4.2]. Using the notation of [1], let

$$J(k, \alpha, M) := \{I(f) \cup \text{range}(f) : f \in G(k, \alpha, M)\}.$$

Then Eq. (3.2) and its proof become valid for $J(k, \alpha, M)$ in place of $I(k, \alpha, M)$. The rest of the theorem holds as stated for $I(k, \alpha, M)$. The following shows the error in (3.2). We have not tried for a best possible result here.

PROPOSITION. *$r_h(I(2, \alpha, M)) \geqslant 1$ for any $\alpha > 0$ and $M > 0$.*

Proof. Let $f(e^{i\theta}) := \frac{1}{2}(1 - \cos\theta)$, $0 \leqslant \theta < 2\pi$. Then f is a C^∞ function from the unit circle S^1 onto $[0, 1]$. Given M and $\alpha > 0$, there is some $\delta > 0$ such that the function $f_1 : z \to \langle \delta f(z), 0 \rangle$ from S^1 into $\mathbb{R}^2$ belongs to $G(2, \alpha, M/2)$. Then $I(f_1)$ is empty. Given any finite set $F \subset]0, \delta[$ and $\epsilon > 0$, there is a C^∞ real function g on $[0, \delta]$ such that $g \geqslant 0$, $g > 0$ on F, $g = 0$ outside F^ϵ, and $g^{(n)}(0) = g^{(n)}(\delta) = 0$ for all n. For a constant $\gamma > 0$ and any $z \in S^1$ let

$$h(z) := \langle \delta f(z), \gamma g(\delta f(z)) \rangle, \qquad \text{Im}\, z \geqslant 0;$$

$$:= \langle \delta f(z), 0 \rangle, \qquad \text{Im}\, z < 0.$$

Then for γ small enough, $h \in G(2, \alpha, M)$. The set of x coordinates of points in $I(h)$ includes F and is included in F^ϵ. Letting $\epsilon \downarrow 0$, the h closure of $I(2, \alpha, M)$ includes the collection of all finite subsets of $]0, \delta[$. Thus for any $\epsilon > 0$, $N(I(2, \alpha, M), \epsilon) \geqslant 2^{(\delta/2\epsilon)-1}$. Again let $\epsilon \downarrow 0$. Hence $r_h(I(2, \alpha, M)) \geqslant 1$.

Q.E.D.

0021-9045/79/060192–02$02.00/0

Note added in proof. On p. 234, line 9, replace 2^k by 3^k; p. 235, line 3, T_x by J_x ; line 7, $2\epsilon^{1/2}$ by $\epsilon^{1/2}$; line 10 up, $-\gamma_k$ by γ_k.

References

1. R. M. Dudley, Metric entropy of some classes of sets with differentiable boundaries, *J. Approximation Theory* **10** (1974), 227–236.
2. R. M. Dudley, Sample functions of the Gaussian process, *Ann. Probability* **1** (1973), 66–103.